ABSTRACT: This work examines the role of working memory capacity in problem solving in
chemistry, and in particular it re-examines the validity of the Johnstone-El Banna predictive model, by
employing non-linear methods. The study correlates the students’ information-processing capacity with
their performance, by using fractal geometry adapted for treating problem-solving data. The rank order
of the subjects’ achievement scores and their working-memory capacities were treated as dynamic
flows and found to possess different geometric characteristics depending on the complexity of the
problem and the method of marking. The classification and interpretation of these characteristics were
made using concepts from complexity theory, such as correlation exponents, fractal dimensions and
entropy. The findings support the hypothesis that long-range correlations exist between the rank order
of the subjects’ achievement scores and their working-memory capacity, and are in agreement with

KEY WORDS: problem-solving; Johnstone-El Banna predictive model; working memory
capacity; mental-demand of a problem; complexity theory; working-memory random walk;
Hurst exponent; long-range correlations; order

INTRODUCTION

The working-memory hypothesis has been of major interest in psychological research after
Miller’s classic article (1956) discussing the capacity limitation in short-term memory. The rich
literature which followed verified the phenomenon and provided cognitive science with a predictive
and explanatory tool. The subject has attracted the interest of science-education researchers as well.
With respect to problem solving in chemistry education, research has shown that, in certain cases,
various psychometric variables [such as working-memory (WM) capacity, mental (or M-space)
capacity, disembedding ability (degree of field dependence-independence), and developmental
level] can be predictive of the student performance (Johnstone & El-Banna, 1986; Niaz & Loggie,
1993; Tsaparlis, Kousathana, & Niaz, 1998). In particular, simple, organic-synthesis problems have
been used to study the necessary conditions for the validity (Tsaparlis, 1998), as well as the
operation and the validity itself (Tsaparlis & Angelopoulos, 1998) of the Johnstone-El Banna
predictive model.

The model states that a student is likely to be: successful in solving a problem if the problem
has a mental demand (M-demand or Z-demand) which is less than or equal to the subject’s WM
capacity ($X$) ($Z \leq X$), but fail for lack of information or recall, and unsuccessful if $Z > X$, unless the student has strategies that enable him/her to reduce the value of $Z$ to become less than $X$. That is, as the problem increases in complexity (in terms of what has to be held and what process has to be performed), there must be a decrease in achievement; moreover, if the holding/thinking space has a finite limit, the decrease of achievement may be rapid after the limit has been reached (Pascual-Leone, 1969; Scandarmalía, 1977). Note that in order for the model to be valid, a number of necessary conditions must be fulfilled (Tsaparlis, 1997; Tsaparlis & Angelopoulos, 2000).

The current work is an attempt to test the WM hypothesis by introducing non-linear methods in the treatment of problem-solving quantitative data. The work correlates the rank order of the subject achievement scores in problem solving with WM capacity and shows how the effect of WM on problem solving can be observed with means and tools of complexity theory.

**RATIONALE AND METHOD**

A problem-solving data set consists of achievements scores. In order to apply complexity theory to this data set, *dynamics* needs to be introduced. This is achieved by using the rank-order sequences of achievement, which are treated as dynamical flows. Rank-order sequences of achievements of the subjects, according to their scores, are generated, and in the place of each subject, his/her score is then replaced by the value of his/her WM capacity. These *dynamic* sequences when treated with tools of complexity theory may appear to possess order or disorder that characterise its fractal geometry. The characterisation of that geometry can be made by using indexes of complexity theory such as the Hurst exponent, fractal dimensions, or entropy.

The working hypothesis which is behind the non-linear treatment is whether there exist scale-invariant, long-range correlations in the created sequence of WMs. The study of these correlations was made by means of the *working-memory random walk* method. This method has previously been reported (Stamovlasis & Tsaparlis, 1999; Tsaparlis & Stamovlasis, 1999).

Data have been taken from the achievement scores in simple organic-synthesis problems (Tsaparlis & Angelopoulos, 2000). The subjects were in the final year of upper secondary school. Seven chemistry problems in organic synthesis of various $Z$-demands, from two to eight, were used. These problems not only exclude numerical or algebraic calculations, but also have a unique chemical logical structure. Note that the problems and the method of administration of the test satisfied the most important necessary conditions for the validity of the problem-solving model (Tsaparlis & Angelopoulos, 2000). Working-memory capacity of the students was assessed by means of the digit span backward (DSB) test.

To test the null hypothesis of randomness, that is if the sequences or series are random, surrogates were developed for each sequence of symbols, by taking the original data set and randomising it completely by shuffling. In this way, the surrogates maintain the same statistical characteristics as the original data set, but with all dynamics being erased. For each sequence the average of twenty surrogates was calculated and standard deviations along with the confidence limits were estimated. The same confidence limits were kept also for the original data. In all cases, none of the surrogates had a higher Hurst exponent than the original sequence. Thus the (non-parametric) level of statistical significance of our results would be at least $p = 1/20 = 0.05$.

Two scores of achievement in each problem have been used for each student. In the first score (*independent marking*), two marks were given for an entirely correct synthesis route; one mark for a correct overall route, but with some minor omissions or errors; and zero marks for wrong
answers. In the second score (accumulated marking), an accumulation of the current mark (according to the first score) with the marks of all problems of lower Z-demand was implemented. Thus, accumulated marking takes into account the history of achievement in all previous problems, and, in addition, it reduces the number of ties in the rank-order positions.

SOME RESULTS AND INTERPRETATION

In the hypothetical one-dimensional space of the rank-order achievement scores, we take a random walk among the subjects with working memory capacity 4 and 5. This walk is named RW 4/5. It is observed that the Hurst exponent for low values of Z-demand, 2, 3, and 4 assumes low values close to the surrogate exponent, which corresponds to theoretical randomness: $H= 0.5$, a normal random walk. The results for the Hurst exponent are shown in Figure 1.

At Z-demand 5, the value of Hurst exponent increases, showing long-range correlations, because subjects with WM capacity 5 outscore subjects with WM capacity 4. When the Z-demand of the problem becomes 6, the problem becomes difficult for everybody, and the sequence becomes disordered again. The same pattern is observed in Figure 2, which is the corresponding diagram for RW5/6. The Hurst exponent increases at Z-demand 6, and decreases after this value.

The accumulated marking led to a slightly different pattern. Figure 3 summarises the above observation showing the increase of Hurt exponent for the three random walks for the accumulated marking case. The increase of Hurt exponent is interpreted as departure from randomness, and occurs at the threshold values of Z-demand, where subjects with a working memory capacity WM = $X + 1$ outscore subjects with WM = $X$.

The existence of threshold Z-demands, where the rank order sequence demonstrates long-range correlations verifies the role of working memory capacity in problem solving. The Hurst exponents for the long-range correlated sequences do not assume very high values, close to unity, or the rank order sequence are not highly ordered, because some subjects with a working memory capacity WM = $X$ manage to outscore subjects with WM = $X + 1$ by employing chunking devices to keep $Z < X$. The educational implications have already been discussed (Tsaparlis & Angelopoulos, 2000).

CONCLUSIONS

This study has provided an alternative verification of the validity of Johnstone-El Banna model. It has been shown, within a non-linear framework, that working-memory capacity correlated with the relative achievement scores in organic-synthesis problem solving. Working-memory random walks in a one-dimensional space of rank-order achievement scores showed characteristics of disorder for problems with low Z-demand, while long-range correlations with a persisted behaviour, characteristic of order, appeared at higher Z-demand values. The change occurred at threshold values $X$ of Z-demand; thus, subjects with WM=$X$ outscored subjects with WM=$X-1$.

A working-memory random walk constitutes a new non-linear method that has been developed for the field of science-education research, which actually deals with non-linear
FIGURE 1. Independent marking case: The Hurst exponent versus Z-demand of the problem for the random walk 4/5. The sudden increase appears from Z-demand 4 to Z-demand 5.

phenomena. In addition a basis for continuing work has been set up, so that random walk methods can be applied for studying the role of other psychometric variables in problem solving.

NOTE: An extended version of this work will appear in the journal *Nonlinear Dynamics in Psychology and Life Sciences* in 2001 (Stamovlasis & Tsaparlis, 2000).

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